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Technical Report
Contract No. N00014-86-K-0742
September 1, 1986 - August 31, 1988

PACKET ERROR PROBABILITIES IN FREQUENCY-HOPPED
SPREAD SPECTRUM PACKET RADIO NETWORKS - MARKOV
FREQUENCY HOPPING PATTERNS CONSIDERED

Submitted to:

Director
Naval Research Laboratory
Washington, D.C. 20375

Attention: Code 2627

Submitted by:

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EECS Department
University of Central Florida

P. Kazakos
Professor

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SCHOOL OF ENGINEERING AND

APPLIED SCIENCE

DEPARTMENT OF ELECTRICAL ENGINEERING

UNIVERSITY OF VIRGINIA

CHARLOTTESVILLE, VIRGINIA 22901

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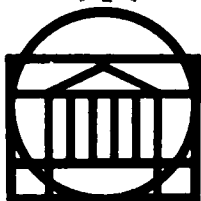
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In this paper we compute the packet error probability induced in a frequency-hopped spread spectrum packet radio network, which utilizes first order Markov frequency hopping patterns. The frequency spectrum is divided into q frequency bins and the packets are divided into M bytes each. Every user in the network sends each of the M bytes of his packet at a frequency bin, which is different from the frequency bin used by the previous byte, but equally likely to be any one of the remaining q-1 frequency bins (Markov frequency hopping patterns). Furthermore, different users in the network utilize statistically independent frequency hopping patterns. Provided that, K users have simultaneously transmitted their packets on the channel, and a receiver has locked on to one of these K packets, we present a method for the computation of

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$p_e(K)$ (i.e. the probability that this packet is incorrectly decoded). Furthermore, we present numerical results (i.e. $p_e(K)$ versus K) for various values of the multiple access interference K, when Reed Solomon (RS) codes are used for the encoding of packets. Finally, some useful comparisons, with the packet error probability induced, if we assume that the byte errors are independent, are made; based on these comparisons, we can easily evaluate the performance of our spread spectrum system.

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1. Introduction

The rapid growth of computer communication has motivated an intense interest in packet switching radio techniques [1]. Furthermore, there is a growing need for computer communication and information distribution in tactical military applications, where spread spectrum waveforms must be used in order to achieve reliable operation in the precense of intentional interference (jamming). As a result, a thorough investigation of spread spectrum packet radio networks is necessary.

The bit error probability induced in frequency hopped spread spectrum systems has been examined before [2]. In [2] the bit error probability is computed for two different models of random frequency hopping patterns. In the first model it is assumed that each random frequency hopping pattern is a sequence of independent random variables (i.e. memoryless frequency hopping patterns), while in the second model it is assumed that each random frequency hopping pattern is a first order Markov chain (i.e. Markov frequency hopping patterns).

The computation of the packet error probability induced in frequency hopped spread spectrum systems, which utilize memoryless frequency hopping patterns has been examined before ([3],[4],[5]). In this paper we are going to compute the packet error probability induced in spread spectrum systems, which utilize Markov frequency hopping patterns. What makes the problem difficult is that the bit errors are not independent. Hence, we cannot extend the results in [2], in a trivial way, in order to compute the packet error

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probability. Nevertheless, some comparisons with the packet error probability induced if we assume that the bit errors are independent are going to be made.

The organization of the paper is as follows. In section 2 we describe the model of our spread spectrum system. Then, in section 3 we present a method to compute the packet error probability induced in our system. In section 4 we utilize the numerical results of section 3 and an educated conjecture to evaluate the performance of our spread specrtum system. Finally, in section 5 we give a brief summary of the accomplishments of the paper, and we discuss some extensions of the model presented in section 2.

2. The Model

The frequency hopping system will now be described. The channel time is divided into slots, and the users in the network initiate their packet transmissions at the beginnings of slots. The frequency spectrum is divided into q frequency bins and the packets are divided into M bytes each. Every user in the network sends each of the M bytes of his packet at a frequency bin , which is different from the frequency bin used by the previous byte, but equally likely to be any one of the remaining q-1 frequency bins (Markov frequency hopping patterns). Furthermore, different users in the network have statistically independent frequency hopping patterns. We also assume that a packet consists of exactly one codeword from a Reed-Solomon (RS) code for which up to e byte errors can be corrected. A packet is declared successfully

transmitted if at most e byte errors occur.

3. A method to compute the packet error probability

Let us assume that K (K≥2) packets are transmitted in the same slot. These packets correspond to K different users in the network. We assign indices to these packets (i.e. packet # 1, packet # 2, ... packet # K). Let us also assume that the receiver locks on to packet # 1. We say that the jth byte of packet # 1 is hit if, during its reception by the receiver, at least one of the other packets (i.e. packets # 2, # 3, ..., # K) occupies the same frequency bin that packet # 1 occupies. Let us now denote by $p_e(K)$, the probability that packet # 1 is decoded incorrectly by the receiver given that K-1 other packets interfere with packet # 1. Our objective is to provide a method for the computation of $p_e(K)$.

We denote by, { f_j^i ; 1≤j≤M }, the frequency hopping pattern corresponding to packet # i (1≤i≤K). In figure 1 we show a realization of the K packet arrivals at the receiver site. It is worth noting that the realization of packet arrivals in figure 1 corresponds to the worst possible case; in other words $p_e(K)$ is maximized when the realization of figure 1 occurs. This is true, because with the realization of figure 1 , during the reception of every byte of packet # 1 , all other K-1 interfering packets are present. We will compute $p_e(K)$ for the realization of packet arrivals depicted in figure 1. Furthermore, we will make the pessimistic assumption, as in [3], that when a byte is hit a byte

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error results.

Let us now denote by, $S(m,n)$; $1 \leq n \leq M$, $m \leq n$, the number of bytes from byte m to byte n of packet # 1 which are in error. Then,

$$p_e(K) = \sum_{i=e+1}^{M} \Pr[S(1,M)=i] \qquad (1)$$

We will compute first the packet error probability , $p_e(K)$, for K=2. In doing so, we will be able to describe the major points of the methodology better. Then, we will discuss the steps required to compute $p_e(K)$ for $K \geq 3$.

Case 1 . K=2.

It is obvious from (1) that, in order to find $p_e(2)$, we must compute the $\Pr[S(1,M)=i]$ for $i=e+1,e+2,\ldots,M$. From the formula of total probability we get:

$$\Pr[S(1,M)=i] = \sum_{s_1^1=1}^{q} \sum_{s_1^2=1}^{q} \Pr(S(1,M)=i/f_1^1=s_1^1, f_1^2=s_1^2) \; \Pr(f_1^1=s_1^1, f_1^2=s_1^2) =$$

$$= \sum_{s_1^1=1}^{q} \sum_{s_1^2=1}^{q} \Pr(S(1,M)=i/f_1^1=s_1^1, f_1^2=s_1^2) \; q^{-2} \qquad (2)$$

We now state a proposition.

Proposition 1. For every $M \geq 1$ and $i \leq M$ the conditional probability, $\Pr(S(1,M)=i/f_1^1=s_1^1, f_1^2=s_1^2)$, depends only on the following two events

$$i) \; s_1^1 = s_1^2 \qquad or \qquad ii) \; s_1^1 \neq s_1^2$$

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In other words $Pr(S(1,M)=i/f_1^1=s_1^1,f_1^2=s_1^2)$ does not depend on the actual values of s_1^1 and s_1^2, as far as, $s_1^1=s_1^2$ or $s_1^1 \neq s_1^2$. Proposition 1 is proven in Appendix A.

Let us now define

$$\underline{f}_j=(f_j^1,f_j^2) \qquad 1 \leq j \leq M \qquad (3)$$

The proof of Proposition 1 is based on the following Lemma.

<u>Lemma 1.</u> The sequence $\{ \underline{f}_j \ ; \ 1 \leq j \leq M \}$ is a Markov chain with stationary transition probabilities.

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Lemma 1 is also proven in Appendix A. Due to proposition 1 equation (2) becomes.

$$Pr[S(1,M)=i]= q^{-1} \ Pr(S(1,M)=i/f_1^1=1,f_1^2=1)+$$
$$+(1-q^{-1}) \ Pr(S(1,M)=i/f_1^1=1,f_1^2=2) \qquad (4)$$

A byproduct of the proof of proposition 1 is that the conditional probabilities, $Pr(S(1,M)=i/f_1^1=1,f_1^2=1)$ and $Pr(S(1,M)=i/f_1^1=1,f_1^2=2)$, satisfy certain recurrent expressions (see Appendix A page 16 for more details). We write these recurrent expressions in the sequel.

$$Pr(S(1,1)=0/f_1^1=1,f_1^2=1)=0 \qquad (5a)$$

$$Pr(S(1,1)=1/f_1^1=1,f_1^2=1)=1 \qquad (5b)$$

$$Pr(S(1,1)=0/f_1^1=1,f_1^2=2)=(q-2)/(q-1) \qquad (5c)$$

$$Pr(S(1,1)=1/f_1^1=1,f_1^2=2)=1/(q-1) \qquad (5d)$$

$$Pr(S(1,n)=0/f_1^1=1,f_1^2=1)=0 \qquad (6a)$$

$;2 \leq n \leq M$

$$Pr(S(1,n)=i/f_1^1=1, f_1^2=1)=(q-1)^{-1} Pr(S(1,n-1)=i-1/f_1^1=1, f_1^2=1)+$$
$$;2\leq n\leq M, \ 0<i\leq n$$
$$+(q-2)(q-1)^{-1} Pr(S(1,n-1)=i-1/f_1^1=1, f_1^2=2) \qquad (6b)$$

$$Pr(S(1,n)=0/f_1^1=1, f_1^2=2)=(q-2)^2(q-1)^{-2} Pr(S(1,n-1)=0/f_1^1=1, f_1^2=2) \quad (7a)$$
$$;2\leq n\leq M$$

$$Pr(S(1,n)=i/f_1^1=1, f_1^2=2)=(q-2)(q-1)^{-2} Pr(S(1,n-1)=i/f_1^1=1, f_1^2=1)+$$
$$;2\leq n\leq M, \ 0<i<n$$
$$+(q-1)^{-1} Pr(S(1,n-1)=i-1/f_1^1=1, f_1^2=2)+$$
$$+(q-2)^2(q-1)^{-2} Pr(S(1,n-1)=i/f_1^1=1, f_1^2=2) \qquad (7b)$$

$$Pr(S(1,n)=n/f_1^1=1, f_1^2=2)=(q-1)^{-1} Pr(S(1,n-1)=n-1/f_1^1=1, f_1^2=2) \quad (7c)$$
$$;2\leq n\leq M$$

Based on expressions (5) through (7) we can compute $Pr(S(1,M)=i/f_1^1=1, f_1^2=1)$ and $Pr(S(1,M)=i/f_1^1=1, f_1^2=2)$ for arbitrary M and $i\leq M$ as follows.

For M=1 all we need are expressions (5). For M>1 we start from n=2 and we evaluate the probabilities $Pr(S(1,n)=i/f_1^1=1, f_1^2=2)$ and $Pr(S(1,n)=i/f_1^1=1, f_1^2=2)$ for i=0,1,...,n based on expressions (6) and (7). Then, we perform similar computations for n=3,4,... up to n=M. Finally we end up having computed the probabilities $Pr(S(1,M)=i/f_1^1=1, f_1^2=1)$ and $Pr(S(1,M)=i/f_1^1=1, f_1^2=2)$ for i=0,1,2,.....M.

Once we have computed $Pr(S(1,M)=i/f_1^1=1, f_1^2=1)$ and $Pr(S(1,M)=i/f_1^1=1, f_1^2=2)$ for i=0,1,2,...,M ,we can find $Pr(S(1,M)=i)$ for i=0,1,2,...,M through formula (4). As a result we can compute $p_e(2)$ via formula (1).

<u>Case 2. $K \geq 3$</u>

The same methodology applies when $K \geq 3$. Hence,

$$Pr[S(1,M)=i]=\sum_{s_1^1=1}^{q}\ldots\sum_{s_1^K=1}^{q} Pr(S(1,M)=i/f_1^1=s_1^1,\ldots,f_1^K=s_1^K) \ q^{-K} \quad (8)$$

Furthermore, a proposition and a Lemma, which are similar to proposition 1 and Lemma 1, can also be stated for every $K \geq 3$. What makes the method more complicated, as K increases, is that the number of events, on which $Pr(S(1,M)=i/f_1^1=s_1^1,\ldots,f_1^K=s_1^K)$ depends on, increases as well. For example, when K=3 the probability $Pr(S(1,M)=i/f_1^1=s_1^1,f_1^2=s_1^2,f_1^3=s_1^3)$ depends on the following four events:

i) $s_1^1=s_1^2=s_1^3$

or ii) $s_1^1=s_1^2$ and $s_1^1 \neq s_1^3$ or $s_1^1 \neq s_1^2$ and $s_1^1=s_1^3$

or iii) $s_1^1 \neq s_1^2$ and $s_1^1 \neq s_1^3$ and $s_1^2=s_1^3$

or iv) $s_1^1 \neq s_1^2$ and $s_1^1 \neq s_1^3$ and $s_1^2 \neq s_1^3$

Nevertheless, once the distinct events, on which the probability $Pr(S(1,M)=i/f_1^1=s_1^1,\ldots,f_1^K=s_1^K)$ depends on, have been correctly identified, recursive expressions, similar to expressions (5) through (7), can also be written for every $K \geq 3$. Consequently, following the same methodology presented in case 1, one can compute $p_e(K)$ for $K \geq 3$.

<u>Numerical Results.</u>

In Table 1 we have included $p_e(2)$ and $p_e(3)$ for q=10,25 and 50 and for the (31,7),(31,15),(31,23),(63,15),(63,31) and (63,47)

Reed Solomon (RS) codes. In the same table, we have also included $\tilde{p}_e(2)$ and $\tilde{p}_e(3)$, where $\tilde{p}_e(K),K\geq2$, corresponds to the packet error probability induced by our spread spectrum system under the assumption that the byte errors of packet # 1 are independent. From table 1 we can make the following observations.

1) $\tilde{p}_e(2)$ is an upper bound of $p_e(2)$ and $\tilde{p}_e(3)$ is an upper bound of $p_e(3)$ for most entries of table 1.

2) $|p_e(2)-\tilde{p}_e(2)|/\tilde{p}_e(2)$ and $|p_e(3)-\tilde{p}_e(3)|/\tilde{p}_e(3)$ are smaller than one for all entries of table 1.

We will see, in the next section, how observation 2 will be helpful in evaluating the performance of our spread specrum system.

Note that if K users are present in a slot then,

$$\tilde{p}_e(K)= \sum_{i=e+1}^{M} \binom{M}{i} [p_b(K)]^i [1-p_b(K)]^{M-i} \quad (10)$$

where $p_b(K)$ is the byte error probability given K. It is easy to show that

$$p_b(K)= 1-(1-2/q)^{K-1} \quad (11)$$

where q is the number of frequency bins in the frequency spectrum.

4. Performance evaluation of the spread spectrum system.

Suppose that the input packet arival process per slot is Poisson with intensity s. Then, let us define (as in [4])

$$q_e(s) = \sum_{K=0}^{\infty} e^{-s} \, s^K/K! \; p_e(K) \qquad (12)$$

to be the average packet error probability induced by our spread spectrum network at input rate s (note that $p_e(0)=p_e(1)=0$). We also define (as in [4]) the maximum average interference level that can be accomodated at a given packet error probability v by

$$s^*(v) = \max\{s: \; q_e(s) \leq v\} \qquad (13)$$

Clearly the value of $s^*(v)$ depends on the number of frequency slots q and the code rate r=k/M (k=number of information bytes, M=total number of bytes in the code). If we account for the expansion of bandwidth induced by q and r, then we can define the normalized maximum average interference level as

$$S^*(v) = r \, s^*(v)/q \qquad (14)$$

Let us also define,

$$\tilde{q}_e(s) = \sum_{K=0}^{\infty} e^{-s} \, s^K/K! \; \tilde{p}_e(K) \qquad (15)$$

(note that $\tilde{p}_e(0)=\tilde{p}_e(1)=0$)

$$\tilde{s}^*(v) = \max\{s: \; \tilde{q}_e(s) \leq v\} \qquad (16)$$

and $\qquad \tilde{S}^*(v) = r \, \tilde{s}^*(v)/q \qquad (17)$

According to observation 2 of the previous section the ratios $|p_e(2)-\tilde{p}_e(2)|/\tilde{p}_e(2)$ and $|p_e(3)-\tilde{p}_e(3)|/\tilde{p}_e(3)$ are smaller than one. We can now state a conjecture.

Conjecture : $|p_e(K)-\tilde{p}_e(K)|/\tilde{p}_e(K) \leq 1$ for $K \geq 4$.

If this conjecture is true, and we find s such that

$$\tilde{q}_e(s) \leq v \qquad (18)$$

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then the same s quarantees that

$$q_e(s) \leq 2v \tag{19}$$

The above discussion indicates that we can use $\tilde{q}_e(s)$ and $\tilde{s}^*(v)$ as reasonable estimates of the performance of our spread spectrum system, provided that the above conjecture is true. It is worth noting that $\tilde{q}_e(s)$ and $\tilde{s}^*(v)$ are easily computable.

5. Conclusions.

We have described (section 3) a method of computing the packet error probability, $p_e(K)$, induced in a spread spectrum system which utilizes first order Markov frequency hopping patterns. We have also computed $p_e(K)$ for K=2 and K=3. Then, we made some comparisons between $p_e(2)$ and $\tilde{p}_e(2)$ and between $p_e(3)$ and $\tilde{p}_e(3)$; $\tilde{p}_e(K)$, K≥2, was defined to be the packet error probability induced by our spread spectrum system if we assume that the byte errors are independent. Based on these comparisons and an educated conjecture we have evaluated the performance of our spread spectrum system (section 4).

The consideration of a slotted channel in section 2 is not so restrictive. The methodology of section 3 and the discussion of section 4 are still valid for the unslotted channel, provided that we resort to an upper bound for the packet error probability. Consider the transmission of a given packet in the unslotted channel, and consider a second situation in which it is assumed that the interference level is constant and equal to the maximum

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number K* of interfering transmissions that take place at any time during the transmission of the given packet in the unslotted system. Clearly, as table 1 also indicates, the packet error probability for the second system is larger than for the first. Hence, $p_e(K*)$ of table 1 will be an upper bound of the packet error probability of the unslotted system.

Furthermore, the pessimistic assumption that a byte hit results in a byte error need not be made either. More optimistic assumptions described in [5] section IV, where thermal noise is also present, can be incorporated in our model too. They will simply make the presentation of section 3 more complicated.

Appendix A

We will first prove Lemma 1. Let us now denote

$$\underline{f}_j = (f_j^1, f_j^2) \qquad ; \ j \geq 1 \qquad (A.1)$$

as in (3). Furthermore, we denote

$$\underline{s}_j = (s_j^1, s_j^2) \qquad ; \ j \geq 1 \qquad (A.2)$$

We will show first that

$$Pr(\underline{f}_j = \underline{s}_j / \underline{f}_{j-1} = \underline{s}_{j-1}, \ldots, \underline{f}_1 = \underline{s}_1) = Pr(\underline{f}_j = \underline{s}_j / \underline{f}_{j-1} = \underline{s}_{j-1}) \quad (A.3)$$

for any $\underline{s}_1, \ldots, \underline{s}_j$ belonging in the state space I (in our case $I = \{ (i_1, i_2): i_1 = 1, \ldots q, \ i_2 = 1, \ldots q \}$) and for any $j \geq 2$.

(A.3) is equivalent to

$$Pr(\underline{f}_j = \underline{s}_j, \ldots, \underline{f}_1 = \underline{s}_1) = Pr(\underline{f}_{j-1} = \underline{s}_{j-1}, \ldots, \underline{f}_1 = \underline{s}_1) \ Pr(\underline{f}_j = \underline{s}_j, \underline{f}_{j-1} = \underline{s}_{j-1})$$
$$[Pr(\underline{f}_{j-1} = \underline{s}_{j-1})]^{-1} \qquad (A.4)$$

But,

$$Pr(\underline{f}_j = \underline{s}_j, \ldots, \underline{f}_1 = \underline{s}_1) = Pr(f_j^1 = s_j^1, f_j^2 = s_j^2, \ldots, f_1^1 = s_1^1, f_1^2 = s_1^2) =$$

independence of $\{f_j^1; j \geq 1\}$ and $\{f_j^2; j \geq 1\}$

$$= Pr(f_j^1 = s_j^1, \ldots, f_1^1 = s_1^1) \ Pr(f_j^2 = s_j^2, \ldots, f_1^2 = s_1^2) =$$

$$= Pr(f_j^1 = s_j^1 / f_{j-1}^1 = s_{j-1}^1, \ldots, f_1^1 = s_1^1) \ Pr(f_{j-1}^1 = s_{j-1}^1, \ldots, f_1^1 = s_1^1)$$
$$Pr(f_j^2 = s_j^2 / f_{j-1}^2 = s_{j-1}^2, \ldots, f_1^2 = s_1^2) \ Pr(f_{j-1}^2 = s_{j-1}^2, \ldots, f_1^2 = s_1^2) =$$

$\{f_j^1; j \geq 1\}$ and $\{f_j^2; j \geq 1\}$ Markov chains

$$= Pr(f_j^1 = s_j^1 / f_{j-1}^1 = s_{j-1}^1) \ Pr(f_j^2 = s_j^2 / f_{j-1}^2 = s_{j-1}^2)$$
$$Pr(f_{j-1}^1 = s_{j-1}^1, \ldots, f_1^1 = s_1^1) \ Pr(f_{j-1}^2 = s_{j-1}^2, \ldots, f_1^2 = s_1^2) =$$

independence of $\{f_j^1; j \geq 1\}$ and $\{f_j^2; j \geq 1\}$

$$= Pr(\underline{f}_{j-1} = \underline{s}_{j-1}, \ldots, \underline{f}_1 = \underline{s}_1) \ Pr(\underline{f}_j = \underline{s}_j, \underline{f}_{j-1} = \underline{s}_{j-1}) / Pr(\underline{f}_{j-1} = \underline{s}_{j-1}) \quad (A.5)$$

The series of steps in (A.5) prove (A.4). The stationarity of $\{\underline{f}_j; j > 1\}$ follows from the fact that

$$Pr(\underline{f}_j = \underline{s}_j / \underline{f}_{j-1} = \underline{s}_{j-1}) = (q-1)^{-2} \qquad ; j \geq 2 \quad (A.6)$$

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(A.5) and (A.6) prove the Lemma. We will now prove proposition 1.

Let us first denote by, $T1(M)$, the following statement.

"The probability, $Pr(S(1,M)=i/f_1^1=s_1^1, f_1^2=s_1^2)$, with $s_1^1=s_1^2$, is independent of the actual values of s_1^1 and s_1^2, for every i such that $0 \leq i \leq M$."

and by, $T2(M)$, the following statement.

"The probability, $Pr(S(1,M)=i/f_1^1=s_1^1, f_1^2=s_1^2)$, with $s_1^1 \neq s_1^2$, is independent of the actual values of s_1^1 and s_1^2, for for every i such that $0 \leq i \leq M$."

We will prove, by induction, that $T1(M)$ and $T2(M)$ are true. For M=1 we have:

$$Pr(S(1,1)=0/f_1^1=s_1^1, f_1^2=s_1^2)=0 \qquad (A.7)$$
$$; s_1^1=s_1^2$$

$$Pr(S(1,1)=1/f_1^1=s_1^1, f_1^2=s_1^2)=1 \qquad (A.8)$$
$$; s_1^1=s_1^2$$

$$Pr(S(1,1)=0/f_1^1=s_1^1, f_1^2=s_1^2)=$$
$$; s_1^1 \neq s_1^2$$

$$= Pr(f_1^1 \neq f_1^2, f_1^1 \neq f_2^2/f_1^1=s_1^1, f_1^2=s_1^2)$$

see figure 1

$$= \sum_{s_2^2=1}^{q} Pr(f_1^1 \neq f_1^2, f_1^1 \neq f_2^2/f_1^1=s_1^1, f_1^2=s_1^2, f_2^2=s_2^2) \; Pr(f_2^2=s_2^2/f_1^1=s_1^1, f_1^2=s_1^2) =$$
$$; s_2^2 \neq s_1^2, \; s_2^2 \neq s_1^1$$

$$= (q-2)/(q-1) \qquad (A.9)$$

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Similarly, we can show that

$$\Pr(S(1,1)=1/f_1^1=s_1^1, f_1^2=s_1^2)=1/(q-1) \quad ; s_1^1 \neq s_1^2 \quad (A.10)$$

(A.7),(A.8),(A.9) and (A.10) indicate that T1(1) and T2(1) are true. Suppose that T1(M) and T2(M) are also true for M=n-1. Let us now denote

$$b_1(i)=\Pr(S(1,n-1)=i/f_1^1=s_1^1/f_1^2=s_1^2) \quad ; 0 \leq i \leq n-1, \ s_1^1=s_1^2 \quad (A.11) \quad \text{and}$$

$$b_2(i)=\Pr(S(1,n-1)=i/f_1^1=s_1^1, f_1^2=s_1^2) \quad ; 0 \leq i \leq n-1, \ s_1^1 \neq s_1^2 \quad (A.12)$$

We will now show that T1(M) and T2(M) are true for M=n. We first note that

$$\Pr(S(1,n)=0/f_1^1=s_1^1, f_1^2=s_1^2)=0 \quad (A.13)$$
$$; s_1^1=s_1^2$$

Furthermore,

$$\Pr(S(1,n)=i/f_1^1=s_1^1, f_1^2=s_1^2)=$$
$$; s_1^1=s_1^2, \ 0<i \leq n$$

$$= \sum_{s_2^1=1}^{q} \sum_{s_2^2=1}^{q} \Pr(S(1,n)=i/f_1^1=s_1^1, f_1^2=s_1^2, f_2^1=s_2^1, f_2^2=s_2^2)$$

$$\Pr(f_2^1=s_2^1, f_2^2=s_2^2/f_1^1=s_1^1, f_1^2=s_1^2)=$$

$$= \sum_{\substack{s_2^1=1 \\ s_2^2=s_2^2 \text{ and } s_2^2 \neq s_1^1}}^{q} \sum_{s_2^2=1}^{q} \Pr(S(1,n)=i/f_1^1=s_1^1, f_1^2=s_1^2, f_2^1=s_2^1, f_2^2=s_2^2) \ (q-1)^{-2}+$$

$$+ \sum_{\substack{s_2^1=1 \\ s_2^2=s_2^2 \text{ and } s_2^2=s_1^1}}^{q} \sum_{s_2^2=1}^{q} \Pr(S(1,n)=i/f_1^1=s_1^1, f_1^2=s_1^2, f_2^1=s_2^1, f_2^2=s_2^2) \ (q-1)^{-2}+$$

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$$+ \sum_{\substack{s_2^1=1 \\ s_2^2 \neq s_2^1 \text{ and } s_2^2 \neq s_1^1}}^{q} \sum_{s_2^2=1}^{q} \Pr(S(1,n)=i / f_1^1=s_1^1, f_1^2=s_1^2, f_2^1=s_2^1, f_2^2=s_2^2) \ (q-1)^{-2} +$$

$$+ \sum_{\substack{s_2^1=1 \\ s_2^2 \neq s_2^1 \text{ and } s_2^2 = s_1^1}}^{q} \sum_{s_2^2=1}^{q} \Pr(S(1,n)=i / f_1^1=s_1^1, f_1^2=s_1^2, f_2^1=s_2^1, f_2^2=s_2^2) \ (q-1)^{-2} =$$

(we cannot have $s_1^1=s_1^2$ and $s_2^2=s_1^1$)

$$= \sum_{\substack{s_2^1=1 \\ s_2^2 = s_2^1 \text{ and } s_2^2 \neq s_1^1}}^{q} \sum_{s_2^2=1}^{q} \Pr(S(2,n)=i-1 / f_1^1=s_1^1, f_1^2=s_1^2, f_2^1=s_2^1, f_2^2=s_2^2) \ (q-1)^{-2} +$$

$$+ \sum_{\substack{s_2^1=1 \\ s_2^2 \neq s_2^1 \text{ and } s_2^2 \neq s_1^1}}^{q} \sum_{s_2^2=1}^{q} \Pr(S(2,n)=i-1 / f_1^1=s_1^1, f_1^2=s_1^2, f_2^1=s_2^1, f_2^2=s_2^2) \ (q-1)^{-2} =$$

$\{\underline{f}_j ; j \geq 1\}$ is a Markov chain (Lemma 1)

$$= \sum_{\substack{s_2^1=1 \\ s_2^2 = s_2^1 \text{ and } s_2^2 \neq s_1^1}}^{q} \sum_{s_2^2=1}^{q} \Pr(S(2,n)=i-1 / f_2^1=s_2^1, f_2^2=s_2^2) \ (q-1)^{-2} +$$

$$+ \sum_{\substack{s_2^1=1 \\ s_2^2 \neq s_2^1 \text{ and } s_2^2 \neq s_1^1}}^{q} \sum_{s_2^2=1}^{q} \Pr(S(2,n)=i-1 / f_2^1=s_2^1, f_2^2=s_2^2) \ (q-1)^{-2} =$$

$\{\underline{f}_j ; j \geq 1\}$ is stationary (Lemma 1)

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$$= \sum_{\substack{s_2^1=1 \\ s_2^2=s_2^1 \text{ and } s_2^2 \neq s_1^1}}^{q} \sum_{s_2^2=1}^{q} \Pr(S(1,n-1)=i-1/f_1^1=s_2^1, f_1^2=s_2^2) \ (q-1)^{-2} +$$

$$+ \sum_{\substack{s_2^1=1 \\ s_2^2 \neq s_2^1 \text{ and } s_2^2 \neq s_1^1}}^{q} \sum_{s_2^2=1}^{q} \Pr(S(1,n-1)=i-1/f_1^1=s_2^1, f_1^2=s_2^2) \ (q-1)^{-2} \qquad (A.14)$$

We now make the following observations.

O.1) If $\underline{f}_1$ is in state (s_1^1, s_1^2) with $s_1^1 = s_1^2$, then $\underline{f}_2$ can be in $q-1$ different states (s_2^1, s_2^2) with the properties $s_2^2 = s_2^1$ and $s_2^2 \neq s_1^1$, independently of the actual values of s_1^1 and s_1^2.

O.2) If $\underline{f}_1$ is in state (s_1^1, s_1^2) with $s_1^1 = s_1^2$, then $\underline{f}_2$ can be in $(q-1)(q-2)$ different states with the properties $s_2^2 \neq s_2^1$ and $s_2^2 \neq s_1^1$, independently of the actual values of s_1^1 and s_1^2.

From (A.14), O.1 and O.2 we conclude that

$$\Pr(S(1,n)=i/f_1^1=s_1^1, f_1^2=s_1^2) = b_1(i-1) \ (q-1)/(q-1)^2 +$$
$$; s_1^1 = s_1^2, \ 0 < i \leq n$$
$$+ b_2(i-1) \ (q-1)(q-2)/(q-1)^2 \qquad (A.15)$$

Similarly, we can show that

$$\Pr(S(1,n)=0/f_1^1=s_1^1, f_1^2=s_1^2) = (q-2)^2/(q-1)^2 \ b_2(0) \qquad (A.16)$$
$$; s_1^1 \neq s_1^2$$

$$\Pr(S(1,n)=i/f_1^1=s_1^1, f_1^2=s_1^2) = (q-2)/(q-1)^2 \ b_1(i) +$$
$$; s_1^1 \neq s_1^2, \ 0 < i < n$$
$$+ (q-1)/(q-1)^2 \ b_2(i-1) + (q-2)^2/(q-1)^2 \ b_2(i) \qquad (A.17)$$

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and,

$$Pr(S(1,n)=n/f_1^1=s_1^1,f_1^2=s_1^2)=(q-1)/(q-1)^2 \ b_2(n-1) \qquad (A.18)$$
$$;s_1^1 \neq s_1^2$$

Expressions (A.15) through (A.18) prove that T1(M) and T2(M) are true for M=n as well. Hence, by induction, T1(M) and T2(M) are true for every M≥1.

Furthermore, the induction procedure above and in particular expressions (A.15) trough (A.18) verified the validity of equations (5) through (7) of section 3.

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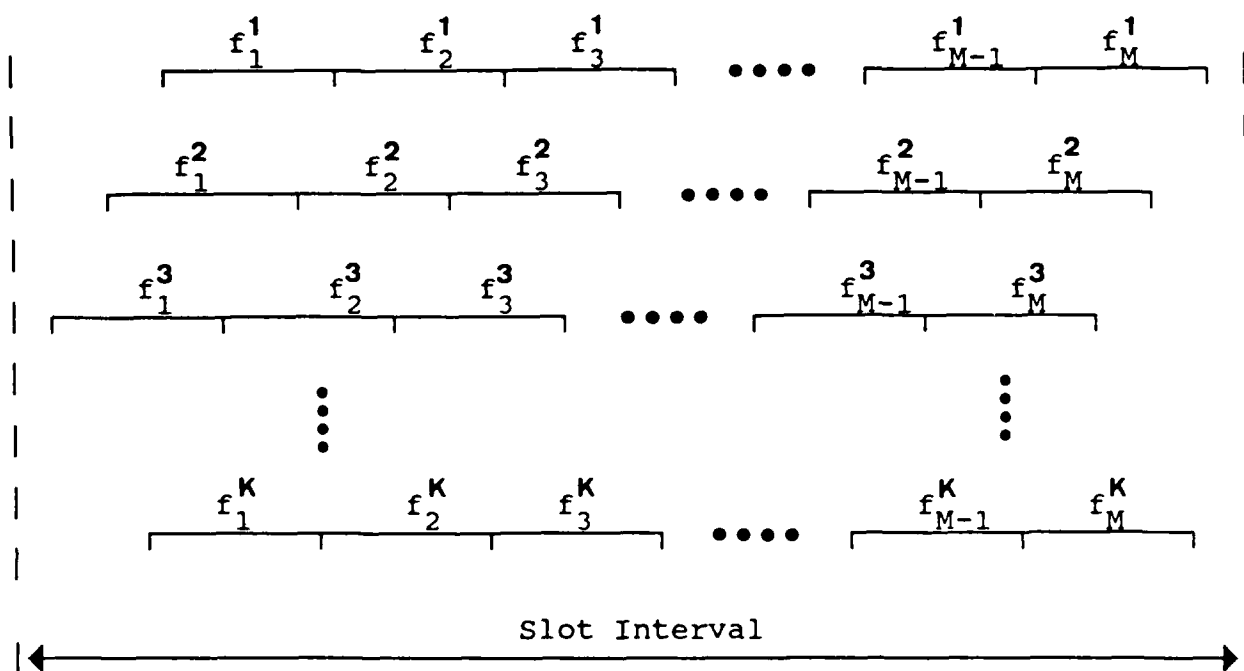_Trans. on Comm._, Vol. COM-30, No. 5, pp. 855-884, May 1982.

$$f_1^1 \quad f_2^1 \quad f_3^1 \quad \bullet\bullet\bullet\bullet \quad f_{M-1}^1 \quad f_M^1$$

$$f_1^2 \quad f_2^2 \quad f_3^2 \quad \bullet\bullet\bullet\bullet \quad f_{M-1}^2 \quad f_M^2$$

$$f_1^3 \quad f_2^3 \quad f_3^3 \quad \bullet\bullet\bullet\bullet \quad f_{M-1}^3 \quad f_M^3$$

$$f_1^K \quad f_2^K \quad f_3^K \quad \bullet\bullet\bullet\bullet \quad f_{M-1}^K \quad f_M^K$$

Slot Interval

Figure 1

Table 1

(31,7) RS code

q	$p_e(2)$	$\widetilde{p}_e(2)$	$|p_e(2)-\widetilde{p}_e(2)|/\widetilde{p}_e(2)$
10	0.78507246	0.77127123	0.17894133D-01
25	0.93451075D-01	0.97636106D-01	0.42863559D-01
50	0.66080673D-02	0.72908919D-02	0.93654467D-01

(31,15) RS code

q	$p_e(2)$	$\widetilde{p}_e(2)$	$|p_e(2)-\widetilde{p}_e(2)|/\widetilde{p}_e(2)$
10	0.13915048	0.15076248	0.77021816D-01
25	0.36450112D-03	0.53081917D-03	0.31332336
50	0.14188960D-05	0.23669732D-05	0.40054412

(31,23) RS code

q	$p_e(2)$	$\widetilde{p}_e(2)$	$|p_e(2)-\widetilde{p}_e(2)|/\widetilde{p}_e(2)$
10	0.25728328D-02	0.43874633D-02	0.41359445
25	0.9370620D-07	0.28409390D-06	0.67015764

Table 1 (continued)

(63,15) RS code

q	$p_e(2)$	$\widetilde{p}_e(2)$	$\lvert p_e(2) - \widetilde{p}_e(2) \rvert / \widetilde{p}_e(2)$
10	0.91745546	0.90631180	0.12295613D-01
25	0.58637826D-01	0.62530221D-01	0.62248220D-01
50	0.74395697D-03	0.87602105D-03	0.15075445

(63,31) RS code

q	$p_e(2)$	$\widetilde{p}_e(2)$	$\lvert p_e(2) - \widetilde{p}_e(2) \rvert / \widetilde{p}_e(2)$
10	0.10016353	0.11197709	0.10549979
25	0.32382994D-05	0.62966015D-05	0.48570678
50	0.11704D-09	0.29755D-09	0.60665434

(63,47) RS code

q	$p_e(2)$	$\widetilde{p}_e(2)$	$\lvert p_e(2) - \widetilde{p}_e(2) \rvert / \widetilde{p}_e(2)$
10	0.10541485D-03	0.26357027D-03	0.6000503
25	0.58D-12	0.445D-11	0.86966292

Table 1 (continued)

(31,15) RS code

q	$p_e(3)$	$\tilde{p}_e(3)$	$|p_e(3)-\tilde{p}_e(3)|/\tilde{p}_e(3)$
10	0.85167393	0.84020289	0.13652702D-01
25	0.35959153D-01	0.39002482D-01	0.78029111D-01
50	0.38309009D-03	0.45761967D-03	0.16286358

(31,23) RS code

q	$p_e(3)$	$\tilde{p}_e(3)$	$|p_e(3)-\tilde{p}_e(3)|/\tilde{p}_e(3)$
10	0.29646470	0.30376087	0.24019453D-01
25	0.25272762D-03	0.35074331D-03	0.27945134
50	0.13704329D-06	0.22482071D-06	0.39043298

(63,31) RS code

q	$p_e(3)$	$\tilde{p}_e(3)$	$|p_e(3)-\tilde{p}_e(3)|/\tilde{p}_e(3)$
10	0.95754921	0.95027964	0.76499271D-01
25	0.10935348D-01	0.12442345D-01	0.12111840
50	0.35134889D-05	0.48095548D-05	0.26947731

(63,47) RS code

q	$p_e(3)$	$\tilde{p}_e(3)$	$|p_e(3)-\tilde{p}_e(3)|/\tilde{p}_e(3)$
10	0.30594667	0.31322755	0.23244998D-01
25	0.14916148D-05	0.26688242D-05	0.44109664
50	0.115D-11	0.286D-11	0.59790209

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